UNIVERSITI TEKNOLOGI MARA
University Publication Centre (UPENA)

ESTEEM

Academic Journal UiTM Pulau Pinang

Volume 6, Number 1 June 2010 ISSN 1675-7939

Esteem Academic Journal is jointly published by the Universiti Teknologi MARA, Pulau Pinang and University Publication Centre (UPENA), Universiti Teknologi MARA, 40450 Shah Alam, Selangor, Malaysia.

ESTEEM

Academic Journal UiTM Pulau Pinang

Volume 6, Number 1 **June 2010** **ISSN 1675-7939**

Foreword

It is indeed a proud moment for the University Publication Centre (UPENA) of UiTM Pulau Pinang for having realised the publication of the sixth volume of the Esteem Academic Journal UiTM Pulau Pinang. In fact, it is the undivided support and all-round commitment from all those who were directly and indirectly involved in this project that was the pivotal factor for this success.

On behalf of UPENA UiTMPP, I would like to, first and foremost, express my sincerest gratitude to Associate Professor Mohd Zaki Abdullah, Director of UiTM Pulau Pinang, Associate Professor Dr Mohamad Abdullah Hemdi, Deputy Director of Academic Affairs and Associate Professor Ir. Damanhuri Jamalludin, Deputy Director of Research, Industry Linkages, Development & Maintenance for their unwavering support and being such a driving force towards this successful endeavour.

Not to be forgotten also is the service rendered by the distinguished panel of external reviewers for their constructive comments and criticisms in ensuring that the papers published in this issue would be of the highest quality. Similarly, the panel of language editors who had worked tirelessly towards ensuring that the papers published were linguistically perfect. To both these groups, UPENA is in awe of your efforts and salutes you!

UPENA is also impressed with the nature of papers submitted for publication. While this issue comprises all engineering based articles, it covers a wide array of sub-engineering disciplines. Kudos to these writers! UPENA sincerely appreciates their efforts and hopes more of our staff will follow in their footsteps.

Finally, research and publication are integral parts of an academic's life at any institution. Apart from being an institutional requirement, it is also essential for our own continuous self-development and knowledge expansion. To this effect, UPENA hopes to play a significant role by providing the platform upon which our staff can realise their dream. So, it is our hope at UPENA UiTMPP that lecturers will take up the challenge and start to publish more vigorously from now on.

Rasaya Marimuthu
Chief Editor
ESTEEM Vol. 6, No. 1, 2010
(Engineering)

Systematic Statistical Approaches in Classifying Physical Properties of Soft Marine Clay

Lim Jit Kheng
Faculty of Civil Engineering
Universiti Teknologi MARA (UiTM), Malaysia
Email: limjitkheng@yahoo.com

Ng Set Foong
Department of Computer and Mathematical Sciences
Universiti Teknologi MARA (UiTM), Malaysia

Mohamad Razip Selamat
School of Civil Engineering
Universiti Sains Malaysia, Malaysia

Eric Goh Kok Hoe
School of Materials and Mineral Resources Engineering
Engineering Campus
Universiti Sains Malaysia, Malaysia

ABSTRACT

In this paper, a rational framework in classifying soil properties using systematic statistical approaches has been proposed and illustrated through a case study using some simple physical index properties of soft marine clay in Peninsular Malaysia. The framework includes an initial data screening, followed by trend analysis, removal of outliers, normality conformance and lastly the descriptive statistics. Common erroneous treatments that used to be overlooked were examined in detail and the results of analyses were interpreted statistically incorporating engineering judgment. The effectiveness of various

ISSN 1675-7939

common statistical methods were compared, contrasted and discussed. The proposed framework was found useful in characterizing soil properties and the important findings were listed.

Keywords: *soft marine clay, physical index property, classification, geotechnical engineering*

Introduction

Soil is known as one of the most complex products by nature and none of the building and civil engineering structures can be constructed without resting on it, directly or indirectly. To achieve a safe and economical structure, the site must be adequately investigated and the properties and behaviors of the soil underneath have to be well-understood or it may jeopardize the project and its environment. However, in real practice, engineers always face with the dilemma of inadequacy in site investigation works due to time and budget constraints. The number of boreholes explored and the samples taken and tested are extremely limited if compared to the volume of soils affected in a project site. This has posed one of the greatest challenges to all the geotechnical engineers in terms of the representativeness of their selected characteristic values for design purposes, or in a broader context, the safety and reliability for the performance of their design.

The nature of geotechnical inherent variability and thus its uncertainty has been treated as a perpetual component in geotechnical design and construction that needs careful evaluation. The probability theory and statistics provide a formal, scientific and quantitative basis in assessing risk and uncertainties, in contrast to the qualitative engineering judgment based on one's past experience. The theory of reliability was first introduced into geotechnical engineering in the early 1970s and has been explored and developed by a number of researchers. Although some applications have been successfully illustrated through years (e.g. Wu, 1974; Tang et al., 1976; Vanmarcke, 1977; Chowdhury et al., 1987; Phoon et al., 1990; Christian et al., 1994; Duncan, 2000; El-Ramly et al., 2002), most areas of geotechnical engineering have not been influenced to any perceptible degree and many geotechnical engineers remained skeptical and reluctant even to make an attempt as reported by National Research Council of United States in year 1995 (NRC, 1995).

In the region of Southeast Asia, the advance probabilistic and reliability theories have been the least deployed in the field of geotechnical engineering even among the researchers. The analytical tools commonly adopted in characterizing soil properties are none other than some simple descriptive functions such as mean, range and simple regression analysis (e.g. Ting and Ooi, 1977; Kobayashi et al., 1990; Mohamad et al., 1994; Chen and Tan, 2003). Some of the reported values were potentially misleading as they were essentially extracted from raw data (soil samples are prone to various disturbances especially those from unsupervised slack site investigation works) without proper statistical treatment.

The soil properties derived from a site investigation are always found widely scattered and limited in number. Consequently, the use of basic statistics to assess these highly variable natural materials may not be as directly applicable as in the case of controllable manufactured materials. It should be noted that the statistical methods used to characterize the soil properties and hence determine its characteristic value have to be exercised with caution. Taking into account the comparable experience of those statistical methods, inappropriate selection of the characteristic value would either result in conservative design (over-design) which is uneconomical or causing various short and long-term serviceability and safety problems (under-design), of which both are undesirable.

Lumb (1966) was among the first to show that most soil properties could be regarded as random variables conforming to the normal or Gaussian theoretical distribution. Consequently numerous statistical methods established based on the normal distribution may safely be applied in estimating design parameters. The statistical functions used in the study to check the normality of natural soil properties were chi-square test (c^2-test) and the graphical plot of standardized normal variate. However, engineers may be puzzled if their soil data are found not normally distributed especially when the number of data is limited.

Phoon and Kulhawy (1999) commented that most of the coefficient of variation (COV) reported in the literature were based on total variability analyses and therefore were considerably larger than the actual inherent soil variability. The error can be attributed to four potential problems: (i) soil data from different geologic units are mixed, (ii) equipment and procedural controls generally are insufficient, (iii) deterministic trend in the soil data are not removed, and (iv) soil data are taken over a long time period. The same could be reasonably inferred from other reported statistical functions especially mean and standard deviation as they are all analysed at one time using similar approach.

Orr (2000) discussed several statistical methods for determining the characteristic values in geotechnical designs in accordance to Eurocode

7. The code defines the characteristic value of a ground property as the value which probability of a worst value governing the occurrence of a limit state is not greater than 5%. In other words, it is a value corresponds to a 95% confidence level that the actual mean value is greater than the selected characteristic value. However, the concept is often misunderstood among the practicing geotechnical engineers and taken as the 5% fractal of a series of derived values obtained from test results.

In this paper, a systematic statistical approach is proposed and illustrated through a case study using some simple physical index properties of soft marine clay in Peninsular Malaysia. The proposed framework includes an initial data screening, followed by trend analysis, removal of outliers, normality conformance and lastly the descriptive statistics. Common erroneous treatments that used to be overlooked or neglected by engineers were examined in details and well taken care. The results of analyses were interpreted statistically with engineering judgment incorporated. The effectiveness of various statistical methods in characterizing soil properties were compared and discussed.

The case study involves a total of 28 undisturbed soil samples extracted from a site investigation report for a selected project carried out in Kemaman, along the east coast of Peninsular Malaysia. A specific Quaternary deposit, commonly known as upper marine clay, from the above profiles was examined covering the layer from 2.5 meter to 12.5 meter below existing grade. Five basic physical index properties of soils, i.e. plastic limit (W_P), natural moisture content (W_N), liquid limit (W_L), bulk unit weight (γ_b) and specific gravity (G_s) were studied and used for illustration.

Data Screening

Disturbance involved in both in situ testing and in sampling and laboratory testing becomes a perpetual component in geotechnical engineering. It may happen during every single step of sampling and handling process including drilling, penetration of sample tube, retrieval, transportation, storage, extraction, trimming, etc. (Orihara et al., 1999). As for the case in Malaysia the effect of disturbance would be worst when the site investigation work was carried out by unskilled labors, not in accordance to specification and without proper supervision as noted by Shaik (2007) and Lim and Ng (2008).

While the responsibility of a site investigation contractor is basically producing a reliable, thorough and accurate factual or descriptive report that is free from interpretation. Meanshile, engineers play the role of cautious interpreters to make the report meaningful and useful in engineering design and construction. In this respect, information on the quality of the data, knowledge about the geological background and other environmental aspects, comparable experience in particular at nearby vicinity and engineering judgment play crucial roles in engineering design and construction.

In this study, an initial data screening process has been incorporated, using fuzzy concepts, to filter out those data points that appear abnormal as compared to the core layer under investigation. This systematic screening process was essential and proved to be extremely useful especially dealing with heterogeneous natural materials that were proned to various disturbances in actual practices. In order to fix the potential anomalies which may bias the outcome of characterization, four criteria established based on the theory of basic soil mechanics have been proposed and elaborated as follows:

Extreme Values

It is rather common to observe erroneous data due to human error (e.g. recording the wrong field in the form, mistakes when transferring data from raw data sheet to the summary table, typing error, etc.) in any data analysis work especially when the data handled are substantial in amount. These erroneous readings are usually extreme, illogical and could be easily detected if they are analyzed cautiously with engineering judgment incorporated. In this set of data, three extreme data points (i.e. W_N = 1.39%, W_N = 6.0% and γ_b = 25.4 kN/m^3) were observed and corrected after refering back to the raw laboratory testing data sheet.

Particle Size Distribution

Based on the British Soil Classification System (BSCS) (BSI, 1981), which is the predominant language of communication between engineers in Malaysia, soil samples will be categorized into coarse- or fine-grained soils based on their particle size distribution at the initial stage. According to the code, soils which have less than 35% of materials that are finer than 0.06 mm (passing through the sieve no. 200 with the aperture size of 63 mm) are classified as coarse soils, and those that have more than

35% are classified as fines. The coarse-grained soils can then be further described according to their grain size distribution, whereas the engineering behavior of fine-grained soils is primarily related to their plasticity.

As coastal deposit is concerned, it is common to have both alluvial sand and alluvial clay found through depth. These layers are usually subdivided for engineering design purpose due mainly to their respective distinct behavior. In this study, the upper marine clays of the depth from 2.5 m to 12.5 m were selected for examination. Though there might have a few samples that had been classified as coarse-grained soils which were originated from localized interbedded sandy material, they were excluded from further analyses. The authors are of opinion that the heterogeneities found do not reflect their actual proportion of existence and should not be counted statistically; rather, they should be only noted in the report especially when their appearance is frequent.

Moisture Content and Atterberg Limits

For cohesive soils, plasticity shows an important effect on engineering properties as shear strength and compressibility. Thus it often appears in correlation study for estimation purpose. The liquid (W_L) and plastic (W_P) limits (referred in group as Atterberg Limits), together with natural moisture content (W_N) are used to reflect the plastic consistency of these fine soils for classification purpose. Other common indices derived from natural moisture content and Atterberg Limits include Plasticity Index (I_P), Liquidity Index (I_L) and Consistency Index (I_C).

The W_N usually lies between W_L (the water content at which the soil ceases to be liquid and become plastic) and W_P (the water content at which the soil ceases to be plastic and become a semi-plastic solid). A qualitative indication of its overconsolidation is generally accepted as follows: (i) If W_N is close to W_L, the soil is normally consolidated, (ii) If W_N is close to W_P, the soil is light to heavily overconsolidated, (iii) If W_N is intermediate, the soil is somewhat overconsolidated, and (iv) If W_N is greater than W_L, the soil is on the verge of being viscous liquid. The soil exists in state (iv) that is easily disturbed and not stable by itself but is held by overburden pressure and interparticle cementation, it is of great potential to create problems (Bowles, 1996). Noted that the same could probably be resulted from disturbance effect due to poor sampling process.

Based on the above understanding, several screening criteria had been derived at to minimize the erroneous data which would bias the result of characterization. The suggested criteria were: (i) $W_P > W_L$

(illogical criterion), (ii) $W_P > W_N$ (illogical criterion), (iii) $W_N > 1.2W_L$ (suggested limiting criterion), and (iv) $W_P < 0.1(W_L + 72)$ (limiting criterion for all soils defined in BSCS). Observation that was found violated with any of the above criteria had to be removed and excluded from data analyses. A total of 2 samples were found not complying with at least one of the above criteria and had been removed.

Reported Typical Ranges

Another screening measure which was found useful and had been adopted in this study was the trimming off suspected values based on reported typical ranges. A critical review had been conducted based on recent works (Abdullah and Chandra, 1987; Balasubramaniam et al., 1989; Kobayashi et al., 1990; Mohamad et al., 1994; Chen and Tan, 2003) done on the engineering properties of soil marine clay in Peninsular Malaysia. The reported ranges; though some overlapped each other to certain extent, while some were found not in agreement. Some of them were too wide which were suspected to have encompassed different homogeneity layers of soils, whilst others were found too narrow in case-specific study due to limited samples accounted.

Cautious examination on the plotted data of the above studies was carried out incorporating comparable experience and engineering judgment amongst the authors. A set of typical ranges for each physical index property has been established and presented in Table 1. A total of 4 samples that fell outside the typical range were excluded from further data analyses. Figure 1 shows the scatter plots of the physical indices against depth for all the observations (before screening). Plasticity chart for the corresponding data is presented in Figure 2.

Trend Analysis

The presence of trend in geotechnical profiles has long been observed and recognized as the systematic physical effects on soils (e.g. overburden stress and stress history). The technique of ordinary least square (OLS) regression was most widely used for trend estimation in geotechnical engineering with polynomial trends up to the order of 2 being predominant and recommended (Lumb, 1974; Asaoka and A-Grivas, 1982; Ravi, 1992; Jaksa et al., 1997; Cafaro and Cherubini, 2002; Uzielli et al., 2005). It is of paramount importance to remove the trend before further analyses if

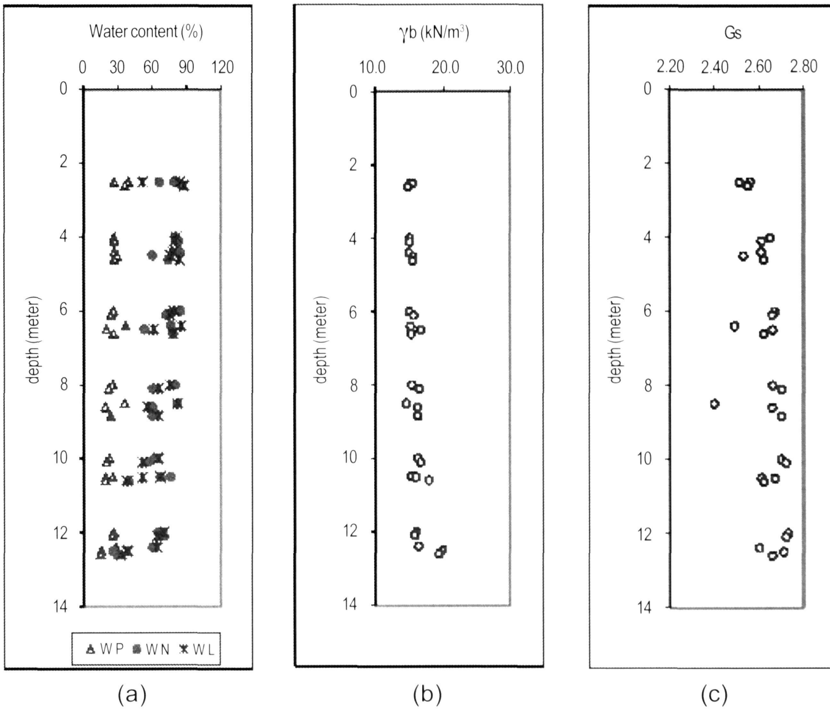


Figure 1: Scatter Plots of the Physical Indices Against Depth (a) Plastic Limit (W_p), Natural Moisture Content (W_N) and Liquid Limit (W_L), (b) Bulk Unit Weight, (γ_b), (c) Specific Gravity (G_s)

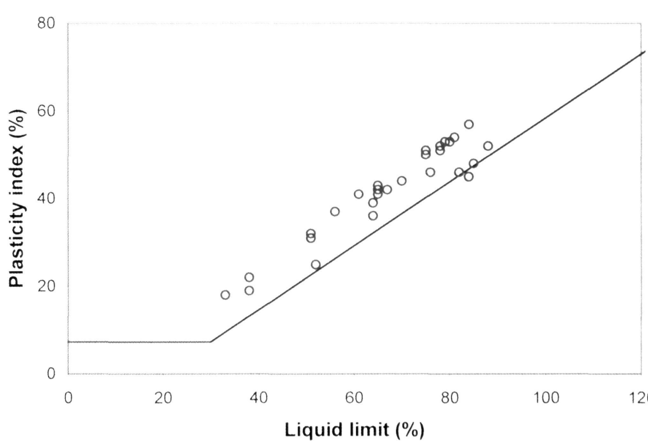


Figure 2: Plasticity Chart

Table 1: Typical Range of the Physical Index of Soft Clay in Peninsular Malaysia

Physical index	Typical range	Number of observations that are out of the typical range	Remark
Plastic limit, W_p (%)	18 – 65	2	15*, 16#
Natural moisture content, W_N (%)	40 – 135	3	39+, 29*, 25#
Liquid limit, W_L (%)	50 – 135	3	38+, 33*, 38#
Bulk unit weight, γ_b (kN/m^3)	13.0 – 18.0	2	19.3*, 20.0#
Specific gravity, G_s	2.45 – 2.75	1	2.40**

Note: The symbols '**', '+', '*' and '#' represent samples no. 18, 24, 28 and 29, respectively

it is found to be significantly present in the data, or else one may obtain a much larger inherent soil variability as compared to the actual.

In this study, low-order polynomial function was formulated to fit the trend of each profile. In particular, linear trend was fitted and its significance was analyzed using OLS regression analysis. The equations of the linear trend for each physical index are given in Table 2. It is noticed that plastic limit, natural moisture content and liquid limit had a negative slope in the equation of the linear trend, which shows that these indices are decreasing with depth. On the other hand, the slopes of bulk unit weight and specific gravity are of positive values, which imply that both bulk unit weight and specific gravity are increasing with depth. These types of trend are generally found in the characteristics of soft clays as agreed with the literatures.

In addition, global F-tests were carried out in order to test the significance of the linear trends. The *p*-values associated with the F-tests are summarized in Table 2. It is noticed that the *p*-values corresponded to the linear trend lines for all the indices were less than 0.05. This implies that the trend lines are all considered significant at 5% level of significance. On the other hand, the coefficient of determination (R^2) corresponded to the linear trends were found ranging from 0.261 (plastic limit) to 0.553 (liquid limit). This again substantiates that these trend lines are of significant. Kerry and Oliver (2004) suggested that raw data could be used only if the R^2 was less than 20% for simplicity reason, or else further analyses could only be done on the residuals after trend had been removed. As a result, the trends were removed and subsequent analyses were carried out using the residuals.

Table 2: Trend in the Profile of the Physical Index of Soft Clay at Kemaman

Physical index	Equation of linear trend	*p*-value of the F-test	Coefficient of determination (R^2)
Plastic limit, W_P (%)	$W_P = 32.8 - 0.87$ Depth	0.015	0.261
Natural moisture content, W_N (%)	$W_N = 84.7 - 1.92$ Depth	0.007	0.309
Liquid limit, W_L (%)	$W_L = 90.0 - 2.46$ Depth	0.000	0.553
Bulk unit weight, γ_b (kN/m^3)	$\gamma_b = 14.7 + 0.13$ Depth	0.002	0.399
Specific gravity, G_s	$G_s = 2.55 + 0.012$ Depth	0.005	0.336

Identification of Outliers

Outliers are data points having anomalous values. They often represent errors in processing or some other spurious effects which are undesirable. Where they are genuine, it is often treated as isolated samples from minor population and generally they are neglected. Extraordinary attention is paid if they are of special interest in certain circumstances. In whatever circumstances, the outliers should be identified and excluded from the characterization of the main population in order to achieve a more representative result of analyses.

This section is in fact a continuation from data screening discussed in section 2 above. The statistical tools illustrated here could serve as the verification process to detect outliers in a more convenient and straightforward way. It should be emphasized however that the identification of outliers using statistical methods alone will not be functioning satisfactorily in view of various anomalies which could not be detected without engineering judgment such as illogical but statistically acceptable value. Two statistical methods which are commonly used to identify the outliers are presented in the following sections.

Mean and Standard Deviation

One of the most convenient ways of identifying outlier is by using mean and standard deviation. Bluman (2004) stated that when the distribution of a data is normal or bell-shaped and data values are beyond 3 standard deviations of the mean, they could be considered as suspected outliers. However, 'mean' is known to be largely affected by extreme low or high values and the results obtained are reliable only when the data is normally distributed. Figure 3 shows the standardized residual plotted against their fitted value for each profile of physical index. It is observed that no outlier was detected (i.e. exceed the magnitude of 3) in any of the profiles based on this criterion.

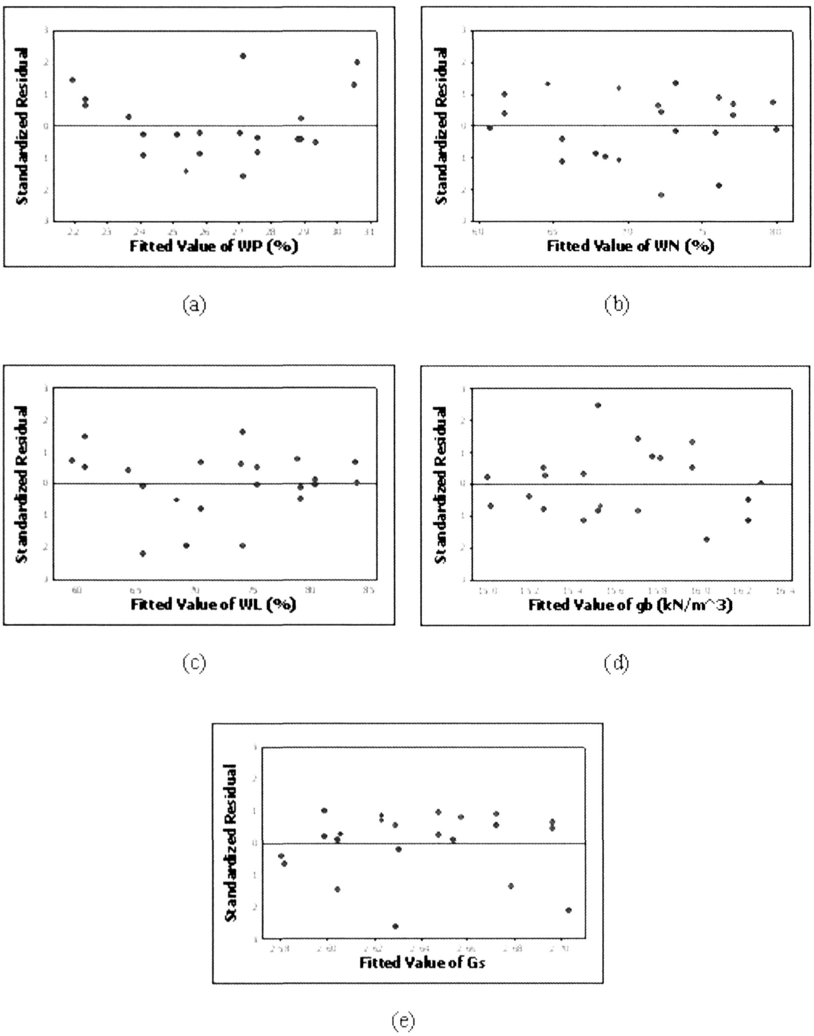


Figure 3: Standardized Residual Plots for Physical Index of Soft Clay (a) Plastic Limit (W_p), (b) Natural Moisture Content (W_N), (c) Liquid Limit (W_L), (d) Bulk Unit Weight, (γ_b), (e) Specific Gravity (G_s)

Box-and-Whisker Plot

Another statistical tool commonly used to identify outlier in scattered data is box-and-whisker plot (also known as boxplot). A boxplot consists of a box where its bottom side is the first quartile while its top side is the third quartile of the data. The median is represented by a horizontal line in between the top and bottom side of the box. The interquartile range (IQR) is the difference between the third and the first quartile. The largest sample value that is no more than 1.5IQR above the third quartile is represented by the whisker end (extended vertical line) on top of the box. The smallest sample value that is no more than 1.5IQR below the first quartile is represented by the whisker end at the bottom of the box. Any point that is larger than 1.5IQR from the first or third quartile is considered as an outlier (represented by '$_*$' in the output). Extreme outlier is any point that is larger than 3IQR from the first or third quartile (represented by 'o' in the output).

As opposed to mean and standard deviation, the statistics used in boxplot are first quartile, median, third quartile and interquartile range, which are less sensitive to the extreme low or high values and the data are not bound to be normally distributed. It provides a more stable and robust approach especially when dealing with widely scattered data such as soil properties. In addition, the graphical presentation of boxplot eases the user in identifying the outlier. Figure 4 presents the boxplots for each physical index of the soft clays. One outlier was detected in the profile of specific gravity (G_s = 2.49 from sample no. 13) and excluded from further analyses. The findings showed that boxplot were sometimes preferred to the 'mean ± 3 standard deviations' approach especially when the data were not distributed normally.

Normality Conformance

Many statistical tests and procedures are based on specific distributional assumptions. The probability distribution of the soil properties is of substantial interest so that generally more powerful parametric techniques can be validated. Various theoretical probability distribution functions have been explored but normal and log normal distributions tend to be widely accepted in geotechnical engineering (Lumb, 1966; DeGroot and Baecher, 1993; Lacasse and Nadim, 1996). In this section, both graphical and numerical approaches are illustrated in investigating whether the

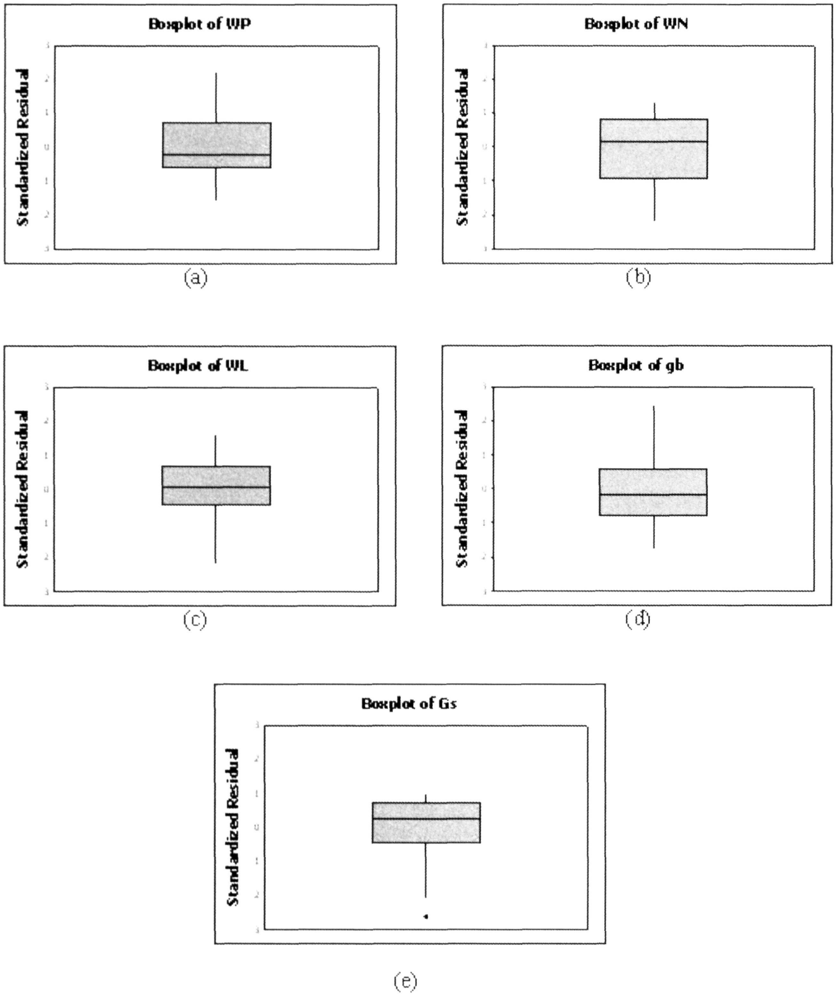


Figure 4: Boxplots Corresponding to the Standardized Residual of Physical Index Properties (a) Plastic Limit (W_p), (b) Natural Moisture Content (W_N), (c) Liquid Limit (W_L), (d) Bulk Unit Weight, (γ_b), (e) Specific Gravity (G_s)

probability distribution of the data is conformed to the most common Normality or Gaussianity criterion.

Graphical Method

Histogram is a useful graphical presentation to visualize the distribution of scattered data such as soil properties. A histogram summarizes the number of observations that fall within specific intervals of value as a vertical bar, thus sometimes called a bar chart. The theoretical normal curve can be superimposed on the histogram to indicate how close the distribution matches to the normal distribution. Many important features are apparent by inspection of this frequency distribution, e.g. the shape, variation, etc. The histogram of each physical index in the form of its residual is given in Figure 5. It is readily observed, for instance, that the distribution of plastic limit was slightly skewed to the right (had a longer right tail) while the distribution of natural moisture content was the other round skewing to the left (had a longer left tail).

In addition, another graphical tool commonly used to diagnose whether the distribution conforms to normality is by using normal probability plot. In normal P-P plot, the cumulative proportion for the variable is plotted against its expected cumulative proportion if the sample were from a normal distribution. If the points cluster around the straight line, the sample is from a normal distribution. A similar plot, called normal Q-Q plot can also be deployed by using the quantiles of the variable rather than its proportion. Figure 6 presents the normal Q-Q plots for the each physical index of soils. It is noticed that some of the points in the normal probability plot of liquid limit and specific gravity, for instances, were deviated from the normal line inferring that the data might not be normally distributed.

Despite of the usefulness of graphical methods in visualizing the probability distribution of the soil properties, interpreting a histogram in a qualitative manner or judging the goodness-of-fit to a line in probability plot to determine its normality conformance is fairly subjective. Therefore, numerical method is usually complemented to verify the graphical outcome in order to assure a more sensible result of analyses.

Numerical Method

The statistical goodness-of-fit tests are usually performed to verify the validity of the specified or assumed distribution model, or diagnosing the normality conformance in this case. Kolmogorov-Smirnov (K-S test) and Anderson-Darling (A-D test) tests are among the most widely used tests and were adopted in the present study. In general, both tests tend

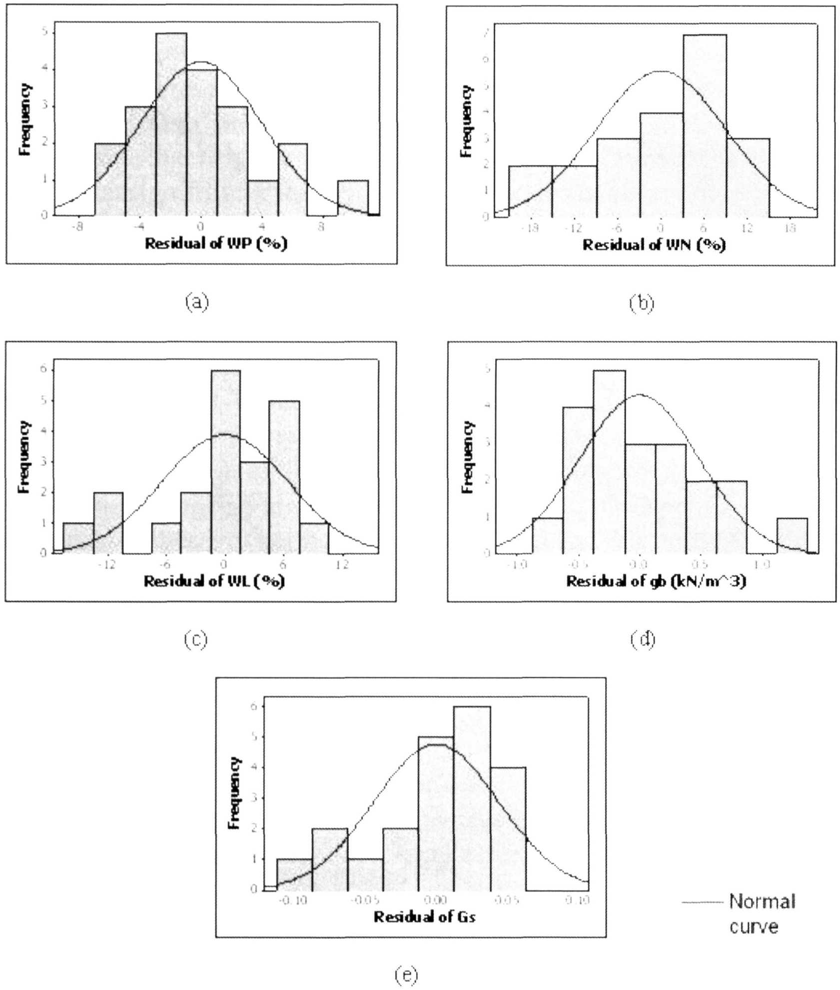
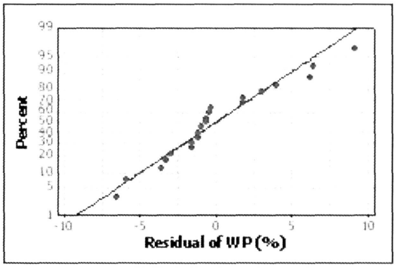


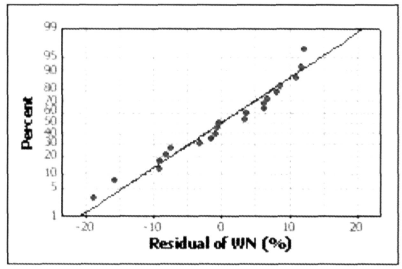
Figure 5: Statistical Distribution of Physical Indices of Soft Clay (a) Plastic Limit (W_p), (b) Natural Moisture Content (W_N), (c) Liquid Limit (W_L), (d) Bulk Unit Weight, (γ_b), (e) Specific Gravity (G_s)

to work well in identifying a distribution as not normal when the distribution under consideration is skewed. On the other hand, both tests are found less discriminating when the underlying distribution is of non-normality due to kurtosis.

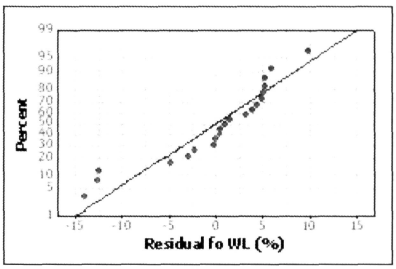
In contrast, A-D test gives more weight to the tails of the distribution than does the K-S test. Therefore, it is more effective in detecting



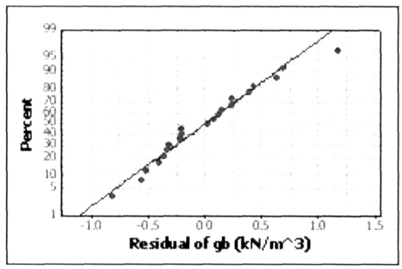
(a)



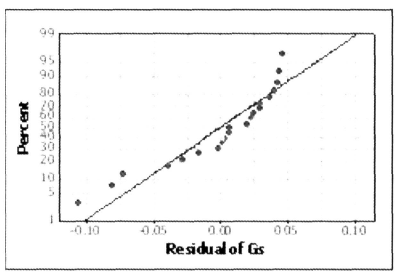
(b)



(c)



(d)



(e)

Figure 6: Normal Probability Plots of Physical Indices of Soft Clay (a) Plastic Limit (W_P), (b) Natural Moisture Content (W_N), (c) Liquid Limit (W_L), (d) Bulk Unit Weight, (γ_b), (e) Specific Gravity (G_s)

departures in the tails of the distribution and particularly useful if the tails are of major concern. K-S test, on the other hand, tends to be more sensitive near the center of the distribution. Furthermore, it is attractive because its distribution does not depend on the underlying cumulative distribution function and it is an exact test which does not depend on an adequate sample size for the approximations to be valid. However, these two tests are restricted to continuous distribution.

The *p*-values associated with the K-S and A-D tests are summarized in Table 3. Based on the defined null hypothesis in these test statistics, any *p*-value that is larger than 0.05 implies that the normality cannot be rejected at significance level of 5%, which is the customary level used in most hypothesis testing. In this study, it is observed that two of the indices (i.e. W_N and γ_b) were found to be unanimously agreed in resembling normal distribution with *p*-values larger than 0.05 and one (i.e. G_s) was agreed by both tests that it was not normal.

However, from the summary in Table 3, it is noticed that the results from both K-S and A-D test statistics did not fully agree with each other. Liquid limit was tested and was found not resembling normal distribution in A-D test (*p*-value = 0.010) but appeared to be normally distributed when tested using K-S test (*p*-value = 0.055) at marginal level. In addition, the null hypotheses of normality for plastic limit was rejected when tested using K-S test (*p*-value = 0.035) but could not be rejected under A-D test (*p*-value = 0.193) at 5% level of significance implying that the distribution can be considered as normal in the case where the tails of the distribution are of major concern, and vice versa. Nonetheless, noted that the violations were not significant if the tests could be tolerated to 1% level of significance.

Identification of the probabilistic distribution for the soil parameter offers very useful information if advanced statistical and probability analyses such as maximum likelihood estimation is to be furthered. To the least, it can justify the validity of using some common powerful parametric techniques which usually are based on normality assumption,

Table 3: Normality Test of Soil Properties

Physical index	*p*-value of the K-S test (1)	*p*-value of the A-D test (2)	Remark
Plastic limit, W_P (%)	0.035	0.193	(2) normal; (1) not
Natural moisture content, W_N (%)	> 0.150	0.382	Both normal
Liquid limit, W_L (%)	0.055	0.010	(1) normal; (2) not
Bulk unit weight, γ_b (kN/m^3)	> 0.150	0.701	Both normal
Specific gravity, G_s	0.029	0.010	Both not normal

Note: Normality test at 5% level of significance.

e.g. t-test to compare means, F-test to compare variances, etc., or else the non-parametric or robust technique may be required.

Descriptive Statistics

Table 4 gives the summarized statistics of the plastic limit (W_P), natural moisture content (W_N), liquid limit (W_L), bulk unit weight (γ_b) and specific gravity (G_s) of soft upper marine clay at a Kemaman site, located along east coast of Peninsular Malaysia. The descriptive statistical functions presented are the number of samples, trend function (represents the mean at different depth), range, standard deviation, variance, mean coefficient of variation (COV), skewness and kurtosis. Another set of descriptive statistics (Table 5) was generated without removing existing trends in the data to show the erroneous results of analyses for comparison purpose. In general, it is learned that the variability of the soil properties listed in Table 5 was consistently considerably higher than those in Table 4.

Noted that where the trend was found significant, the mean value should not be represented by a constant value but to use a well-fitted low-order polynomial function as proper description as illustrated in this case. The error was expected to be even more significant as the gradient of the trend was higher and where the soil layer under investigation was getting thicker. The predicted means from the generated trend functions did not only agree well with the literature, but also furnished with other related important information for the reader to justify its representativeness and validity.

The dispersion of the observations was described by the magnitude of sample standard deviation or variance. It can be seen from the scatter plots in Figure 1 or referring to the histograms shown in Figure 5 that the dispersion of W_P was the smallest while the W_N appeared to be the largest among the three measurement of water content of soils. A more meaningful interpretation could be appreciated where the same parameter was being compared between one site and another. Logically, the one that was widely dispersed possessed higher uncertainty and risk than the concentrated thereby should be treated with larger margin of safety in the design. This is the principal concept in reliability-based design as opposed to the conventional limit states approach. The standard deviation for W_P, W_N, W_L, γ_b and G_s were 3.97%, 8.95%, 6.44%, 0.48 kN/m^3 and 0.044, respectively, whereas their respective variance were 15.79(%)^2,

Table 4: Summary Statistics of Soil Properties

Physical index	No. of samples	Trend function	Range	Standard deviation	Variance	Mean COV	Skewness	Kurtosis
Plastic limit, W_P (%)	21	W_P = 32.0 – 0.83 Depth	15.5	3.97	15.79	0.14	0.60	0.11
Natural moisture content, W_N (%)	21	W_N = 84.4 – 1.91 Depth	31.0	8.95	80.10	0.13	-0.54	-0.50
Liquid limit, W_L (%)	21	W_L = 89.2 – 2.41 Depth	23.9	6.44	41.42	0.10	-1.03	0.54
Bulk unit weight, γ_b (kN/m^3)	21	ã$_b$ = 14.7 + 0.13 Depth	2.0	0.48	0.23	0.03	0.57	0.30
Specific gravity, G_s	21	G_s = 2.56 + 0.012 Depth	0.16	0.044	0.002	0.02	-1.24	0.86

Table 5: Summary Statistics of Soil Properties (Erroneous Results without Removing Trend – FOR COMPARISON ONLY)

Physical index	No. of samples	Mean	Range	Standard deviation	Variance	COV	Skewness	Kurtosis
Plastic limit, W_P (%)	21	26.0	20.0	4.74	22.50	0.18	1.24	2.39
Natural moisture content, W_N (%)	21	70.6	33.0	10.75	115.65	0.15	-0.07	-1.54
Liquid limit, W_L (%)	21	71.7	37.0	9.91	98.21	0.14	-0.31	-0.66
Bulk unit weight, γ_b (kN/m^3)	21	15.6	2.0	0.62	0.39	0.04	0.27	-1.33
Specific gravity, G_s	21	2.64	0.20	0.057	0.003	0.02	-0.36	-0.61

$80.10(\%)^2$, $41.42(\%)^2$, $0.23(kN/m^3)^2$ and 0.002. Standard deviation is usually preferred to variance in describing the dispersion of the measurement because the former had the same unit as the original data.

Another more convenient measure of dispersion is given by the coefficient of variation (COV), obtained by normalizing the standard deviation with respect to the mean of the observations. COV is a dimensionless quantity and perceived as almost a universal parameter in representing the variability of soil property. It is readily observed that the COV for a given soil property can be rather stable from soil to soil, as opposed to COV relating to different properties of the same soil (Bourdeau and Amundaray, 2005). Note that if the variable resembles the theoretical normal distribution, a COV of 0.30 implies that the variable can encompass 90 – 100% of its fractal, lower or higher than the mean value. This very large range has to be taken with cautious to ensure that it is representative of the inherent variability of the soil property at which it is expected. The COV for W_P, W_N, W_L, γ_b and G_s obtained in this study were 0.14, 0.13, 0.10, 0.03 and 0.02, respectively (Table 4). The results agreed well with the reported values and showed an even smaller magnitude of variability due to its 'within-site' study in this case.

Skewness and kurtosis are commonly used to characterize the shape of the probability distribution in conjunction to the plot of histogram. From Table 4, it is observed that the values of skewness and kurtosis were all very close to zero (a normal distribution has skewness and kurtosis value of zero). The skewness for W_P and γ_b were found positive in values implying that their distributions were slightly skewed to the right (had a slightly long right tail) whilst the rest (i.e. W_L, W_N, and G_s) were of negative which are slightly skewed to the left (had a slightly long left tail). On the other hand, the values of kurtosis obtained were all positive except for natural moisture content and this was an indication that most of the physical indices have slightly longer tail and shorter peak than normal distribution.

Conclusion

A rational framework in characterizing soil properties using systematic statistical approaches had been proposed and illustrated through a case study using some simple physical index properties of soft marine clay in Peninsular Malaysia. The framework included an initial data screening, followed by trend analysis, removal of outliers, normality conformance and lastly the descriptive statistics. Proper statistical treatments had been exercised and the results of analyses were interpreted both statistically and from engineering perspective. The effectiveness of several statistical methods commonly used in characterizing soil properties were compared, contrasted and discussed. The proposed framework was found useful and the important findings are listed as follows:

i. An initial data screening using fuzzy concepts with engineering judgment incorporated appears to be very useful forming an essential process when dealing with heterogeneous natural materials that are proned to various disturbances which sometimes can not be treated satisfactorily using pure statistical methods.
ii. The presence of trend, especially when it is apparently significant, has to be removed before further analyses can be done in order to ensure the outcome of characterization is representative with nominal inevitable errors.
iii. Even if the outliers are strongly believed to be genuine, they should be treated as isolated samples from minor population but not to be included in the characterization of the major population in most circumstances.
iv. Many statistical tests and procedures are based on specific distributional assumptions, and thus the probability distribution of the soil properties should be identified so that the techniques adopted can be validated. Noted that the parametric techniques are usually more powerful than the non-parametric.
v. At least two different techniques should be performed on any single test to complement each other's finding especially when the test outcomes are probabilistic or not absolute in nature. Moreover, graphical approaches though not objective, often give a better visualization of the important features.
vi. As far as geotechnical engineering is concerned, engineering judgment and comparable experience should always be prevailed and incorporated in the quantitative statistical approaches so that the representative and appropriate characteristic values could be derived without prejudice.

References

Abdullah, A.M.L.B., and Chandra, S. (1987). Engineering properties of coastal subsoils in Peninsular Malaysia. *Proceedings of the Ninth Southeast Asian Geotechnical Conference*. Bangkok, Thailand, Dec 7-11.

Asaoka, A., and A-Grivas, D. (1982). Spatial variability of the undrained strength of clays. *Journal of the Geotechnical Engineering Division*, ASCE, 108(GT5), 743-756.

Balasubramaniam, A.S., Phien-Wej, N., Indraratna, B., and Bergado, D.T. (1989). Predicted behaviour of a test embankment on Malaysian marine clay. *Proceedings of the International Symposium on Trial Embankments on Malaysian marine clays*. Kuala Lumpur, Nov 6-8.

Bluman, A.G. (2004). *Elementary Statistics*. Boston: McGraw-Hill.

Bourdeau, P.L., and Amundaray, J.I. (2005). Non-parametric simulation of geotechnical variability. *Geotechnique*, 55(2), 95-108.

Bowles, J.E. (1996). *Foundation Analysis and Design*. 5th Ed., Singapore: McGraw-Hill Book Co.

BSI. (1981). *BS5930: 1981: Code of Practice for Site Investigations*. British Standard Institution, London.

Cafaro, F., and Cherubini, C. (2002). Large sample spacing in evaluation of vertical strength variability of clayey soil. *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 128(7), 558-568.

Chen, C.S., and Tan, S.M. (2003). Some engineering properties of soft clay from Klang area. *Proceedings of the Second International Conference on Advances in Soft Soil Engineering and Technology*. Putrajaya, Jul 2-4.

Chowdhury, R., Tang, W.H., and Sidi, I. (1987). Reliability model of progressive slope failure. *Geotechnique*, 37(4), 467-481.

Christian, J.T., Ladd, C.C., and Baecher, G.B. (1994). Reliability applied to slope stability analysis. *Journal of Geotechnical Engineering*, ASCE, 120(12), 2180-2207.

DeGroot, D.J., and Baecher, G.B. (1993). Estimating autocovariance of in-situ soil properties. *Journal of Geotechnical Engineering*, ASCE, 119(1), 147-166.

Duncan, J.M. (2000). Factors of safety and reliability in geotechnical engineering. *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 126(4), 307-316.

El-Ramly, H., Morgenstern, N.R., and Cruden, D.M. (2002). Probabilistic slope stability analysis for practice. *Canadian Geotechnical Journal*, 39, 665-683.

Jaksa, M.B., Brooker, P.I., and Kaggwa, W.S. (1997). Inaccuracies associated with estimating random measurement errors. *Journal of Geotechnical and Geoenvironmental Engineering,* ASCE, 123(5), 393-401.

Kerry, R., and Oliver, M.A. (2004). Average variograms to guide soil sampling. *International Journal of Applied Earth Observation and Geoinformation*, 5, 307-325.

Kobayashi, Y., Todo, H., Weerasinghe, W.A.Y., and Chandra, P. (1990). Comparison of coastal clays found in Singapore, Malaysia and Indonesia. *Proceedings of the Tenth Southeast Asian Geotechnical Conference*. Taipei, Apr 16-20.

Lim, J.K., and Ng, S.F. (2008). A survey on the planning and implementation of site investigation. *Esteem Academic Journal UiTM Pulau Pinang*, 4(1), 123-141.

Lumb, P. (1966). The variability of natural soils. *Canadian Geotechnical Journal*, 3(2), 74-97.

Lumb, P. (1974). Applications of statistics in soil mechanics. *In* Soil mechanics – new horizons, I.K. Lee, ed., Newnes-Butterworths, London, p. 44-111.

Mohamad, R., Chin, C.H., Ismail, M.P., and Zainal, Z.M. (1994). The engineering geology and geotechnics of soft deposits in Peninsular Malaysia. *Geotropika 94*, Universiti Teknologi Malaysia.

NRC. (1995). *Probabilistic Methods in Geotechnical Engineering*. Board on Energy and Environmental System, National Research Council, Washington.

Orihara, K., Chandra, P., and Yokoi, Y. (1999). Site investigation and in-situ testing. *Proceedings of the 17th Conference of ASEAN Federation of Engineering Organizations*, Singapore, Nov 20.

Orr, T.L.L. (2000). Selection of characteristic values and partial factors in geotechnical designs to Eurocode 7. *Computers and Geotechnics*, 26, 263-279.

Phoon, K.K., and Kulhawy, F.H. (1999). Characterization of geotechnical variability. *Canadian Geotechnical Journal*, 36, 612-624.

Phoon, K.K., Quek, S.T., Chow, Y.K., and Lee, S.L. (1990). Reliability analysis of pile settlement. *Journal of Geotechnical Engineering*, ASCE, 116(11), 1717-1735.

Ravi, V. (1992). Statistical modelling of spatial variability of undrained strength. *Canadian Geotechnical Journal*, 29, 721-729.

Shaik, A.W. (2007). Towards more reliable site investigation information. *Proceedings of the 16th South-East Asian Geotechnical Conference*, 8-11 May 2007, Kuala Lumpur.

Tang, W.H., Yucemen, M.S., and Ang, A.H.-S. (1976). Probabilistic-based short term design of soil slopes. *Canadian Geotechnical Journal*, 13, 201-215.

Ting, W.H., and Ooi, T.A. (1977). Some properties of coastal alluvial of Peninsular Malaysia. *International Symposium on Soft Clay*, Bangkok. 89-100.

Uzielli, M., Vannucchi, G., and Phoon, K.K. (2005). Random field characterisation of stress-normalised cone penetration testing parameters. *Geotechnique*, 55(1), 3-20.

Vanmarcke, E.H. (1977). On the reliability of earth slopes. *Journal of the Geotechnical Engineering Division*, ASCE, 103(GT11), 1247-1265.

Wu, T.H. (1974). Uncertainty, safety and decision in soil engineering. *Journal of the Geotechnical Engineering Division*, ASCE, 100(GT3), 329-348.